

Stabilizing the Runaway Quiver in Supergravity

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Abstract

We study stabilizations of the supersymmetry breaking runaway quiver in string embeddings. Calculations are performed in four dimensional effective supergravity. Constraints on closed string fields in a type IIA construction are given. The particular case of stabilization by stringy instanton effects in a type IIB model is considered.

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1 Introduction

It has been recently observed that fractional branes at singularities can give rise to quiver gauge theories that dynamically break supersymmetry. In [1], the construction consisted of a cone over dP_1 and N D5 branes at the end of the duality cascade with gauge group $U(3N) \times U(2N) \times U(N)$. It was argued that at the end of the cascade, confinement leads to deformation of the complex structure of the geometry; obstruction to such deformation causes supersymmetry breaking. A similar effect was conjectured for $Y^{p,q}$, $p > q > 0$ and higher del Pezzos with the initial number of D5 branes chosen carefully. In [4], analysis was done in the regime where $U(3N)$ dominates, with the conclusion that the dynamically generated ADS superpotential drives the system away from the supersymmetric point at the origin of moduli space. A similar analysis was done for the case of $Y^{p,p-1}$ in [11] (see also [5], [6], [32], [7], [8], [9], [10], [13]).

Supersymmetry breaking by this method has been used to engineer gauge mediation in string theory [12]. The standard model is realized using fractional branes on a partially collapsed dP_8 , the supersymmetry breaking sector consists of branes at a collapsed dP_1 , and open strings stretching between the two stacks act as messengers.

However, in [14], Intriligator and Seiberg show by a detailed field theory analysis that the models proposed above have a runaway direction in field space, and thus do not actually break supersymmetry in the desired manner. In [15], dimer technology was used to study the infrared behavior of the entire $Y^{p,q}$ family and some examples of $L^{a,b,c}$ singularities, and arguments in favor of such runaway behavior were given.

Various constructions using open string fields have been made to stop this runaway and produce string-phenomenological models [10], [16], [17], [18], [19]. In [10], for example, extra fields were added to the model in the form of light massive flavors. By choosing a specific set of D7 branes, D3-D7 states were made to couple with the fields of the D3 brane at the dP_1 singularity. The resulting extended quiver produced a long-lived metastable supersymmetry breaking vacuum. Higher del Pezzos have been treated in [18], while applications to gauge mediation have been discussed in [16]. Various other applications and extensions have been discussed in [20], [21], [22], [23], [24], [25], [26], [27], [28], [29].

In this paper we consider supergravity stabilizations of the runaway quiver coming from closed string effects. Closed string effects come from Calabi Yau moduli in realistic embeddings of the supersymmetry breaking quiver in string theory. Such embeddings have been done in both type IIA [30] and type IIB [31].

At the level of the string embedding of the quiver, the runaway behavior in field space comes from the lack of proper moduli stabilization mechanisms. In the type IIA case, moduli stabilization is performed by RR and NS flux [46], [33], [34], [37], [38], [39], [40], [41], [42]. Consistent orientifolding and the Freed Witten anomaly cancellation condition introduce various constraints on the Calabi Yau and the quiver locus. In this paper, we couple stabilized closed string moduli with the open string sector, and perform an effective four dimensional supergravity analysis for a variety of toy models. The conclusion is that under mild conditions on the Kahler potential and with proper choices of flux or instanton contributions to the superpotential, the quiver gauge theory is indeed stabilized. Comments on the possible uplift to dS vacua are made. One expects these basic features to be true in a full-blown IIA computation.

In the case of embedding in type IIB, Kahler moduli stabilization comes from instanton effects [45]. Embedding of the runaway quiver has been performed in [31], where various instanton effects have been explicitly calculated. We study this string realization in detail, and perform a supergravity analysis to show that the system stabilizes in a certain regime of calculability.

The plan of the paper is as follows. In section 2, we summarize the quiver gauge theory results in field theory. In section 3, we work out general supergravity stabilization conditions, and apply them in a type IIA scenario. In section 4, we discuss stabilization in a type IIB construction, treating the example of [31].

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2 The Runaway Quiver : Field Theory Description

The gauge theory of M D5 branes on the complex cone over F_1 is given by $SU(3M) \times SU(2M) \times SU(M)$. For the purpose of this paper, we consider the case $M = 1$. The various fields transform as follows [14]:

	$SU(3M)$	$SU(2M)$	$SU(M)$	$[SU(2)$	$U(1)_F$	$U(1)_R]$
Q	$\mathbf{3M}$	$\overline{\mathbf{2M}}$	$\mathbf{1}$	$\mathbf{1}$	1	-1
\bar{u}	$\overline{\mathbf{3M}}$	$\mathbf{1}$	\mathbf{M}	$\mathbf{2}$	-1	0
L	$\mathbf{1}$	$\mathbf{2M}$	$\overline{\mathbf{M}}$	$\mathbf{2}$	0	3
L_3	$\mathbf{1}$	$\mathbf{2M}$	$\overline{\mathbf{M}}$	$\mathbf{1}$	-3	-1

(2.1)

The gauge invariant fields are defined as

$$Z = \det_{fj} Q^f \bar{u}_j, \quad X_{ia} = Q \bar{u}_i L_a, \quad V^a = \frac{1}{2} L_b L_c \epsilon^{abc}. \quad (2.2)$$

The low energy spectrum of the system consists of the fields V^i , $i = 1, 2$ and $V^3 \equiv V$ after all other fields have satisfied their SUSY equations of motion. The dynamical superpotential is

$$W = 3 (V \Lambda_3^7)^{1/3}. \quad (2.3)$$

The Kahler potential far out in V moduli space is given by

$$K_{eff} \approx K_{cl} = 2\sqrt{T} = 2\sqrt{VV^\dagger + V^i V^{i\dagger}} \quad (2.4)$$

This leads to a runaway in field space of the form:

$$V_{eff} \approx 2|\Lambda_3^7|^{2/3} (VV^\dagger)^{-1/6} \quad (2.5)$$

with $V_i = 0$.

3 Stabilization Conditions and Type IIA examples

In embeddings of the above quiver gauge theory in type IIB string theory, the runaway in field space is caused by a lack of moduli stabilization mechanism at the string level. Closed string Kahler moduli are typically stabilized by instanton effects. However, in the present context, such instantons develop extra zero modes due to their interaction with the fractional branes, which can lead to cancellations in the effective superpotential. Some progress has been made recently in that direction, for example in [31], where instanton effects are explicitly calculated. In the next section, we consider stabilizations in such a scenario.

An alternate embedding begins with the observation that such quiver gauge theories occur at non-geometric phases in the Kahler moduli space, and hence can be treated in a type IIA scenario by using mirror symmetry. Supergravity methods can be used in the mirror picture. A full blown IIA embedding of the runaway quiver consists of generic NS and RR flux stabilizing complex structure and Kahler moduli of the Calabi Yau Y respectively. The quiver is realized by D6 branes wrapping special Lagrangian cycles. In compact models, an orientifold projection is introduced. A number of conditions on the quiver locus and the geometry of the IIB mirror Calabi Yau X have to be imposed - (i) X should contain a pair of disjoint del Pezzos (S, S') which don't intersect the fixed point locus X^σ of the orientifold projection and (ii) the holomorphic involution of the orientifold projection should be compatible with the large complex structure limit in the complex structure moduli space of X , so that computations can be done in the supergravity limit of the mirror IIA scenario.

In the mirror IIA construction, the superpotential gets the following flux contributions

$$W_K = \int_Y F \wedge e^{-J_Y}, \quad W_H(x^k, t_\lambda) = -2x^k g_k - it_\lambda h^\lambda \quad (3.1)$$

where W_K is the superpotential contribution to the Kahler moduli of Y , J_Y is the Kahler class of Y , F is the RR flux and is given by $F = F_0 + F_2 + F_4 + F_6$, W_H is the superpotential contribution to the complex structure moduli of Y , (g_k, h^λ) are the NS flux, and x^k, t_λ are $h_+^3 = h^{2,1} + 1$ holomorphic coordinates on the $N = 1$ complex structure moduli space [43], [44]. The coordinates x^k and t_λ are given by

$$x^i = 1/2 \int_Y \Omega_Y^c \wedge \beta^i, \quad t_\lambda = \int_Y \Omega_Y^c \wedge \alpha_\lambda \quad (3.2)$$

where $(\alpha_\lambda, \beta^i)$ form a symplectic basis of three-cycles on Y and Ω_Y^c is a linear combination of the RR three-form $C^{(3)}$ and the real part of the holomorphic three-form of Y . A specific choice of symplectic basis for explicit calculations demands more constraints on the construction [30] - (iii) the natural push-forward maps $H_2(S) \rightarrow H_2(X)$ and $H_2(S') \rightarrow H_2(X)$ have rank one, and (iv) under the orientifold projection, the anti-invariant subspace $H_-^{1,1}(X)$ is one-dimensional and is spanned by the difference $S - S'$ between the divisor classes of the conjugate del Pezzos S and S' .

In general, W_K is enough to stabilize all Kahler moduli. On the other hand, NS flux is subject to the Freed-Witten anomaly cancellation condition, which can hinder moduli stabilization by hindering generic flux. Requiring F-flatness of the superpotential then requires additional conditions due to the non-appearance of certain complex structure moduli due to the anomaly cancellation condition. In [30], explicit embeddings of the quiver gauge theory have been constructed taking into account all the above constraints in the case of certain quintic threefolds.

In this section, we perform an effective four-dimensional supergravity analysis of the quiver. We take general closed string contributions to the Kahler potential and superpotential, and couple them to the open string sector. Our strategy is to begin with a supersymmetric vacuum on the closed string side, and stabilize the open string field $\psi = \kappa^2 V_3$ in that vacuum. A self-consistent analysis is performed for $\psi \ll 1$, which allows independent stabilization of the closed string sector and removes higher order corrections to the open string sector coming from $U(1)$ D-terms. Comments on the possible uplift to dS vacua are made.

We work out specific examples for the case of a single complex modulus x in a type IIA context, without taking into account the complications introduced by the Freed Witten anomaly. The Kahler potential is taken to be a power series in x , while the superpotential is considered to be either a flux contribution like (3.1), or a typical instanton effect.

Our general result is that in the case of a flux superpotential, tuning the value of flux enables stabilization in the region of calculability and possible uplift to small positive cosmological constant. In the case of an instanton superpotential, consistent stabilization without strong constraints on the Kahler potential or superpotential requires a hierarchy of scales between the two sectors. Uplift to dS vacuum is correspondingly more difficult to achieve. The stabilization procedure in both cases puts mild conditions on the Kahler potential, and in the second case, on the instanton contribution. It is expected that in a full IIA calculation, these basic features would be maintained.

3.1 General Analysis

We take the following Kahler potential and superpotential:

$$\kappa^2 K = (\psi \bar{\psi})^{1/2} (1 + \gamma \sum p_i) + \sum f_i \quad (3.3)$$

$$W = \Lambda^3 \psi^{1/3} (1 + \sigma \sum q_i) + \Lambda_1^3 \sum g_i \quad (3.4)$$

where $p_i = p(x_i, \bar{x}_i)$, $f_i = f(x_i, \bar{x}_i)$, $q_i = q(x_i)$, $g_i = g(x_i)$, the x_i being an arbitrary number of closed string moduli. We consider the simple case where different closed string moduli x_i and x_j are decoupled. γ and σ parametrize the strength of the coupling between the open and closed string sectors in the Kahler potential and superpotential respectively. In particular, we will be working to first order in these parameters. Also, $\psi = \kappa^2 V_3 \ll 1$, so that the field V_3 is stabilized below the Planck scale.

The Kahler metric may be inverted, and to first order in γ one obtains

$$K^{\psi \bar{\psi}} = 4\kappa^2 (1 - \gamma \sum p_i) |\psi|$$

$$K^{x_i \bar{\psi}} = (1/2) \kappa^2 \gamma \partial_i p_i (\partial_i \bar{\partial}_i f_i)^{-1} \psi$$

$$K^{x_i \bar{x}_i} = \kappa^2 [(\partial_i \bar{\partial}_i f_i)^{-1} - \gamma (\partial_i \bar{\partial}_i p_i) (\partial_i \bar{\partial}_i f_i)^{-2} |\psi|] \quad (3.5)$$

while $K^{x_i \bar{x}_j}$, $i \neq j$ starts at order γ^2 .

All the contributions to the supergravity scalar potential can be computed, and we keep terms upto order $|\psi|^{1/3}$. The resulting stabilization places constraints on the functions f_i, g_i, p_i, q_i . Generally, the potential is of the form

$$V = e^{\kappa^2 \Sigma f} \left[A |\psi|^{1/3} + B |\psi|^{-1/3} \right] + V_{\text{closed}} \quad (3.6)$$

where A, B can be expressed in terms of f_i, g_i, p_i, q_i .

The non-Abelian D-term contributions to the potential are set to zero by working on the D-flat moduli space defined by (2.2). The $U(1)$ D-term contributions in general introduce new open-closed mixing terms, since the gauge coupling is a holomorphic function of the closed string moduli. However, these mixings begin at order $|\psi|$, and we neglect them.

We study some limiting cases of the parameters γ and σ , and work out some examples with a single complex structure modulus. The functions f and p in the Kahler potential are taken to be power series expansions in the

complex structure modulus. The superpotential term is taken to be a flux contribution or a typical instanton contribution.

We note that similar supergravity calculations have been performed (see [35], [36], for example) in the context of uplifting the KKLT AdS vacuum by coupling it to a SUSY breaking sector such as an O’Raifeartaigh or ISS model.

3.2 $\gamma = \sigma = 0$

In this case, one obtains

$$A = \sum_i \kappa^2 \Lambda^3 \Lambda_1^3 \left[(2/3) \Sigma \bar{g} + [\partial_i f_i \bar{\partial}_i \bar{g}_i + \partial_i f_i \bar{\partial}_i f_i \Sigma \bar{g}] / \partial_i \bar{\partial}_i f_i - 3 \Sigma \bar{g} \right] e^{i\theta/3} + c.c.$$

$$B = (4/9) \kappa^2 \Lambda^6 \quad (3.7)$$

Here, θ is the phase of ψ . We work out the case of a single complex structure modulus x , with $f = f_0 + \alpha_1(x + \bar{x}) + \alpha_2(x\bar{x}) + \dots$

(i) Taking a single complex structure modulus, we have $g = g_0 x$. For $|\psi| \ll 1$ we can stabilize the closed string sector independently. A stable supersymmetric solution is located at

$$\begin{aligned} \text{Re} x_{\min} &= (\text{Re} \alpha_1 / 2\alpha_2) \left[-1 \pm \sqrt{1 - (4\alpha_2 / \alpha_1^2)} \right], \\ \text{Im} x_{\min} &= -(\text{Im} \alpha_1 / \text{Re} \alpha_1) \text{Re} x_{\min}. \end{aligned} \quad (3.8)$$

Stabilizing the open string sector, one obtains

$$|\psi|_0^{1/3} = [(4/9)(\Lambda / \Lambda_1)^3]^{1/2} g_0^{-1/2} J^{-1/2} \quad (3.9)$$

where

$$J = (4/3 - \alpha_1^2 / \alpha_2)(\text{Re} x_{\min}) - \text{Re} \alpha_1 / \alpha_2 - 4\alpha_1 x_{\min}^2 g_0 > 0. \quad (3.10)$$

Note that (3.10) places constraints on the coefficients appearing in the Kahler potential. Minimization with respect to phases has been done, and for simplicity we have assumed $\text{Im} \alpha_1$ is small. The self-consistency condition $|\psi| \ll 1$ can be obtained by tuning the flux g_0 to be large, without assuming a hierarchy of scales between Λ and Λ_1 . On the other hand, the value of the potential at the minimum is

$$V_{\min} = 2e^{\kappa^2 K} \kappa^2 \Lambda^{9/2} \Lambda_1^{3/2} g_0^{1/2} J^{1/2} - 3\kappa^2 e^{\kappa^2 K} g_0^2 x_{\min}^2 \Lambda_1^6. \quad (3.11)$$

Tuning the flux such that $J \sim g_0 x_{\min}^2$, one can potentially lift to a dS vacuum.

On the other hand, assuming a hierarchy of scales $\Lambda/\Lambda_1 \ll 1$ without tuning the flux automatically satisfies $|\psi| \ll 1$, but in this case uplift to a dS vacuum is difficult to achieve.

(ii) Taking a typical instanton correction to the superpotential sets $g = \beta e^{-\alpha x}$. For $|\psi| \ll 1$, the stabilization of the closed string sector is decoupled from the open string sector. We start with a stable closed string vacuum satisfying $D_x W = 0$, located at

$$x_{\min} = (1/\alpha_2)(\alpha - \bar{\alpha}_1) \quad (3.12)$$

For small $\text{Im} x$, minimizing with respect to the phases sets $(\theta/3 + \alpha \text{Im} x) = \pi$. Minimizing the open string sector with respect to $|\psi|$ sets

$$|\psi|_0^{1/3} \sim (4/21)^{1/2} (\Lambda/\Lambda_1)^{3/2} \beta^{-1/2} \exp[(\alpha/2\alpha_2)(\alpha - \alpha_1/2 - \bar{\alpha}_1/2)] \quad (3.13)$$

The condition $|\psi| \ll 1$ can be achieved by having $\beta \gg 1$ and $\Lambda \ll \Lambda_1$.

At the minimum, we obtain

$$V \sim (112/27)^{1/2} \kappa^2 \Lambda^3 (\Lambda \Lambda_1)^{3/2} \beta^{1/2} \exp[f_0 - (\alpha - \alpha_1)^2/\alpha_2] \times \\ \times \exp[(3\alpha/2\alpha_2)(\alpha - \alpha_1/2 - \bar{\alpha}_1/2) - 3\kappa^2 \Lambda_1^6 \beta^2 \exp[f_0 - (\alpha - \alpha_1)^2/\alpha_2]] \quad (3.14)$$

In the regime of calculability $|\psi| \ll 1$, the vacuum remains close to the closed string AdS vacuum, and there isn't much uplift.

3.3 $\gamma = 0, \sigma \neq 0$

In general, apart from flux contributions to the superpotential, instanton corrections coming from the closed string sector can couple to the open string fields. In that case, $\sigma \neq 0$, $q = \beta e^{-\alpha x}$. Such corrections will also lead to open-closed coupling in the Kahler potential, but as a limiting case we set $\gamma = 0$ here. We take the case of a single IIA complex structure modulus and consider two cases - where the pure closed string contribution g is a flux effect, and where g is also due to an instanton effect.

In section 4, we study a type IIB embedding scenario where such couplings have been explicitly calculated.

For $\gamma = 0, \sigma \neq 0$, we obtain

$$\begin{aligned}
A = \sum_i \kappa^2 \Lambda^3 \Lambda_1^3 & \left[(\sigma \bar{\partial}_i \bar{g}_i \partial_i q_i) / \partial_i \bar{\partial}_i f_i + (2/3) \Sigma \bar{g} + [\partial_i f_i \bar{\partial}_i \bar{g}_i (1 + \sigma \Sigma q) + \right. \\
& \left. + \sigma \bar{\partial}_i f_i \partial_i q_i \Sigma \bar{g}] / \partial_i \bar{\partial}_i f_i + (1 + \sigma \Sigma q) \partial_i f \bar{\partial}_i f \Sigma \bar{g} / \partial_i \bar{\partial}_i f_i - 3(1 + \sigma \Sigma q) \Sigma \bar{g} \right] e^{i\theta/3} + c.c. \\
B = (4/9) \kappa^2 \Lambda^6 (1 + \sigma \Sigma q) & \quad (3.15)
\end{aligned}$$

(i) We consider a flux contribution to the superpotential as before $g = g_0 x$ and take $q = \beta e^{-\alpha x}$. For $|\psi| \ll 1$ and σ such that $|\psi|^{-1/3} \sigma \ll 1$, the open-closed mixing in the potential is small, and the closed string sector can be stabilized independently as before. We obtain a supersymmetric minimum, where the value of x is given by (3.8). Stabilization on the open string side gives ψ as a function of the coefficients $\alpha, \alpha_1, \alpha_2$. For small x , this simplifies and we get

$$|\psi|^{1/3} = (2/3)(\Lambda/\Lambda_1)^{3/2} g_0^{-1/2} [\alpha_2(1 + \sigma\beta)]^{1/2} [\sigma\beta\alpha - (1 + \sigma\beta)\text{Re}\alpha_1]^{-1/2}. \quad (3.16)$$

The calculability condition can be satisfied by taking large values of g_0 . We also note that reality of $|\psi|$ sets the condition $\sigma\beta\alpha - (1 + \sigma\beta)\text{Re}\alpha_1 > 0$. The value of the potential at the minimum is given by

$$\begin{aligned}
V_{\min} = 2e^{\kappa^2 K} \kappa^2 \Lambda^{9/2} \Lambda_1^{3/2} g_0^{1/2} [\alpha_2(1 + \sigma\beta)]^{1/2} [\sigma\beta\alpha - (1 + \sigma\beta)\text{Re}\alpha_1]^{1/2} - \\
- 3\kappa^2 \Lambda_1^6 e^{\kappa^2 K} g_0^2 x_{\min}^2. \quad (3.17)
\end{aligned}$$

In principle, it is possible to uplift the AdS vacuum by controlling the flux g_0 such that $(g_0^{3/2} x_{\min}^2)^{-1} \sim 1$.

(ii) We now consider the case where the pure closed string contribution to the superpotential is also an instanton effect. In this case, $g = \beta_g e^{-\alpha x}$, $q = \beta_q e^{-\alpha x}$. The supersymmetric minimum of the closed string sector is given by (3.12). For small x , the open string sector is stabilized at

$$|\psi|^{1/3} = (2/3)(\Lambda/\Lambda_1)^{3/2} [1 + \sigma\beta_q]^{1/2} J^{-1/2}, \quad (3.18)$$

where $J = \beta_g[7/3 + (\alpha\alpha_1/\alpha_2) - \alpha_1^2/\alpha_2] + \beta_g\beta_q[3\sigma + 2\sigma\alpha\alpha_1/\alpha_2 - \sigma\alpha^2/\alpha_2 - \sigma\alpha_1^2/\alpha_2] > 0$ is a condition that can be satisfied if $\alpha > \alpha_1$, for example. Also, $|\psi| \ll 1$ requires the hierarchy of scales $\Lambda/\Lambda_1 \ll 1$.

As in (3.14), the minimum of the system remains close to the AdS .

3.4 $\gamma \neq 0, \sigma = 0$

In the limit where open and closed string contributions may be taken to be decoupled in the superpotential, the Kahler potential of the system will in general still contains couplings between the two sectors. Considering $\gamma \neq 0, \sigma = 0$, we get

$$A = \sum_i \kappa^2 \Lambda^3 \Lambda_1^3 \left[(2/3) \Sigma \bar{g} + [\partial_i f_i \bar{\partial}_i \bar{g}_i + \partial_i f_i \bar{\partial}_i f_i \Sigma \bar{g}] / \partial_i \partial_i f_i - 3 \Sigma \bar{g} - \right. \\ \left. - (2/3) \gamma \partial_i \bar{g}_i \partial_i p_i / \partial_i \bar{\partial}_i f_i \right] e^{i\theta/3} + c.c$$

$$B = (4/9) \kappa^2 \Lambda^6 (1 - \gamma \Sigma p) \quad (3.19)$$

(i) We take $g = g_0 x$, $f = f_0 + \alpha_{1f}(x + \bar{x}) + \alpha_{2f}(x\bar{x}) + \dots$, and $p = p_0 + \alpha_{1p}(x + \bar{x}) + \alpha_{2p}(x\bar{x}) + \dots$. The closed string sector is stabilized at the supersymmetric vacuum given by (3.8). For $x \rightarrow 0$, the open string sector is stabilized at

$$|\psi|^{1/3} = (2/3)(\Lambda/\Lambda_1)^{3/2} g_0^{-1/2} [1 - \gamma p_0]^{1/2} [2/3 \gamma \alpha_{1p}/\alpha_{2f} - \alpha_{1f}/\alpha_{2f}]^{-1/2}. \quad (3.20)$$

This gives the constraint $2/3 \gamma \alpha_{1p} > \alpha_{1f}$. As before, $|\psi| \ll 1$ can be achieved by $g_0 \gg 1$, while an uplift of the AdS vacuum can be achieved by tuning g_0 such that $(g_0^{3/2} x_{\min}^2)^{-1} \sim 1$.

(ii) For an instanton-like contribution $g = \beta e^{-\alpha x}$, the closed string supersymmetric minimum lies at (3.8), while for small x , the open string field is stabilized at

$$|\psi|_0^{1/3} = (4/3)(\Lambda/\Lambda_1)^{3/2} \beta^{-1/2} (1 - \gamma p_0)^{1/2} J^{-1/2} \quad (3.21)$$

where $J = (7/3) - (\alpha_{1f}^2/\alpha_{2f}) + \alpha \alpha_{1f}/\alpha_{2f} - (2/3) \gamma \alpha \alpha_{1p}/\alpha_{2f}$. We require $J > 0$.

4 Stabilization with Stringy Instantons in IIB

Following [31], we consider the quiver gauge theory on a singular dP_1 geometry, with added Euclidean D3 brane instantons. The D3's which intersect the singularity will in general also give rise to Ganor strings stretching from the occupied nodes of the quiver. Denoting quiver fields generically by ψ_i , the superpotential of the system is deformed by effects of the form

$$\Delta W \sim f(\psi_i) \exp(-\text{Vol}) \quad (4.1)$$

where Vol is the volume of the D3. The scalar potential can be stabilized to obtain metastable vacua.

Concretely, the complex cone over dP_1 can be described in terms of toric data as follows. The non-trivial two-cycles in dP_1 are denoted by f and C_0 . A basis of branes is given by

$$[\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4] = [\mathcal{O}_{F_1}, \mathcal{O}_{F_1}(C_0 + f), \overline{\mathcal{O}_{F_1}(f)}, \overline{\mathcal{O}_{F_1}(C_0)}]. \quad (4.2)$$

Denoting the \mathbb{P}^1 fibrations over f and C_0 by D_2 and D_3 , and the the dP_1 base by D_5 , one obtains the nonzero triple intersections

$$D_5^3 = 8, \quad D_5 D_2 D_3 = 1, \quad D_5^2 D_2 = D_5 D_3^2 = -1. \quad (4.3)$$

Various instanton effects can be calculated in this geometry. This requires knowledge about the topology of the D3 brane and its spectrum of Gannor strings. The Euclidean D3's and the quiver nodes wrap a surface S on the del Pezzo cone, and carry different line bundles \mathcal{L}_A and \mathcal{L}_B over S . The most general bundle for the instanton is $X_{ab} = \overline{\mathcal{O}_{D_5}(aC_0 + bf)}$. Computing the number of fermionic zero modes between X_{ab} and $\mathcal{L}_{1,2,3}$ gives

$$n_{\text{ferm}}(X_{ab}, \mathcal{L}_{1,2,3}) = (a + 2b, \quad -3 + a + 2b, \quad 2 - a - 2b). \quad (4.4)$$

An important instanton effect one can have in this geometry is the Affleck-Dine-Seiberg (ADS) instanton effect. In this case, $a = 0$, $b = 1$; that is, the instanton wraps the bundle \mathcal{L}_3 . It turns out that the instanton contribution in this case leads to the superpotential

$$W_{ADS} = \frac{\Lambda^7}{Z} e^{-S_1}. \quad (4.5)$$

Here, S_1 is given by

$$\text{Re}(S_1) = (1/2)(8r_5^2 - r_3^2 - 2r_3r_5 - 4r_2r_5 + 2r_2r_3) + r_3 - 2r_5. \quad (4.6)$$

where r_5, r_2, r_3 parametrize the Kahler form J in terms of the toric data in the following way:

$$J = r_5 D_5 + r_2 D_2 + r_3 D_3. \quad (4.7)$$

Volumes are measured in string units $\alpha' = (2\pi)^{-1}$.

One can have stringy deformations of the above field theory superpotential in the case $b > 1$ or $b \leq -1$. The superpotential in this case is

$$W_{\text{stringy}} = \frac{\Lambda^7}{M_s^6} V_3 \sum_{b>1 \& b \leq -1} f(b) e^{-S_1 + (b-1)S_2}. \quad (4.8)$$

where S_2 is given by

$$\text{Re}(S_2) = 3r_3 - 2r_2 \quad (4.9)$$

and W_{stringy} is valid near the quiver locus $|\text{Re}(S_2)| \ll 1$.

Apart from these contributions to the superpotential, there is the usual term W_{flux} responsible for fixing complex structure moduli, and $W_{\text{gaugino}} \sim \Lambda_{SO(8)} e^{-S_3}$ arising from gaugino condensation in pure $SO(8)$ gauge theory on a divisor D_6 at infinity. D_6 does not intersect D_5 , and thus there is no mixture between quiver fields and instanton effects in W_{gaugino} . Here, α is a number less than one, and S_3 is given by

$$\text{Re}(S_3) = r_2 r_3 - (1/2)r_3^2. \quad (4.10)$$

The superpotential is a sum of all these effects. Denoting $x_a = 2\text{Re}(S_a)$, the regime of validity of this superpotential is

$$x_3 \gg x_1 \gg 1, \quad |S_2| \ll 1, \quad \text{or equivalently,} \quad r_2 \sim (3/2)r_3 \gg r_5 \gg 1 \quad (4.11)$$

To simplify the analysis of the vacuum structure of this system, we set $r_2 = (3/2)r_3$, and only consider the contribution from instantons with $b = 1$. The superpotential of the system, after integrating out the fields Z and X_{ia} , is

$$W_{\text{eff}} = W_{\text{flux}} + 3\Lambda^{7/3} \kappa^{-2/3} \psi_3^{1/3} e^{-S_1/3} + \Lambda_{SO(8)}^3 e^{-\alpha S_3} \quad (4.12)$$

where

$$\psi_a = \kappa^2 V_a, \quad \kappa^2 = M_{pl}^{-2} \quad (4.13)$$

In our regime of validity, we can use the standard large radius expression for the Kahler potential:

$$\kappa^2 K = -2\log\left(f_1 + f_2 \sqrt{\psi_a \psi_a^*}\right). \quad (4.14)$$

where f_1 is the volume of the threefold, and f_2 is the volume of the divisor D_5 , in string units.

Under our approximations, we obtain f_1 and f_2 in terms of the fields x_1 and x_3 as follows:

$$f_1 = (1/4\sqrt{2})x_3^{1/2}x_1^{1/2}[x_3^{1/2} - x_1^{1/2}] \quad (4.15)$$

and

$$f_2 = (1/2)x_1 \quad (4.16)$$

Equipped with W and K , we have the supergravity scalar potential

$$V = \exp(\kappa^2 K) \left(K^{i\bar{j}} W_{eff;i} W_{eff;\bar{j}}^* - 3\kappa^2 W_{eff}^* W_{eff} \right) + \frac{1}{2g_X^2} \sum_{a=1}^3 (D_a)^2 \quad (4.17)$$

where the $U(1)$ D-terms are given by:

$$D_1 = -D_2 = -2 \left(\psi^a \partial_{\psi^a} K + \partial_{x_1} K \right), \quad D_3 = 0.$$

First, we perform an analysis to minimize V with respect to the fields ψ_1 and ψ_2 . Since these fields do not appear in the superpotential or its derivatives, their contribution to the F-term comes from the inverse Kahler metric and derivatives of the Kahler potential. We study the region of field space where $\alpha_1 = \psi_1 \bar{\psi}_1 \ll \psi_3 \bar{\psi}_3$ and $\alpha_2 = \psi_2 \bar{\psi}_2 \ll \psi_3 \bar{\psi}_3$. V_F as a function of α_1 and α_2 takes the form:

$$V_F(\alpha_1, \alpha_2) = \kappa^2 \left(\frac{J_1 \alpha_1 + J_2 \alpha_2 + J_3 \alpha_1 \alpha_2 + J_4}{J_5 \alpha_1 + J_6 \alpha_2 + J_7 \alpha_1 \alpha_2 + J_8} \right) - 3\kappa^2 W \bar{W}. \quad (4.18)$$

The J_i are functions of $\psi_3 \bar{\psi}_3$ and S_1, S_3 . In writing the J_i , we have used the approximation $\psi_a \bar{\psi}_a \sim \psi_3 \bar{\psi}_3$. We see that J_1, J_2, J_3, J_4 have mass dimension six, and consist of products of W and its derivatives. In the limit of $W_{\text{flux}} \gg W_{\text{correction}}$ where $W_{\text{correction}} = 3\Lambda^{7/3} \kappa^{-2/3} \psi_3^{1/3} e^{-S_1/3} + \Lambda_{SO(8)}^3 e^{-\alpha S_3}$, we can write

$$V_F(\alpha_1, \alpha_2) \sim \kappa^2 W_{\text{flux}}^2 \left(\frac{J_1 \alpha_1 + J_2 \alpha_2 + J_3 \alpha_1 \alpha_2 + J_4}{J_5 \alpha_1 + J_6 \alpha_2 + J_7 \alpha_1 \alpha_2 + J_8} - 3 \right). \quad (4.19)$$

where $J_i, i = 1$ to 4 have been redefined, and are now dimensionless.

On the other hand, the D-term contribution is

$$V_D = \kappa^{-4} g^{-2} [J_9 \alpha_1 + J_{10} \alpha_2 + J_{11}]^2 \quad (4.20)$$

For $g^2\kappa^6 W_{\text{flux}}^2 \ll 1$, the D-term dominates over the F-term, and we can argue that the potential is minimized at $\alpha_1 = \alpha_2 = 0$. Since the F-term contribution is essentially monotonic as a function of α_1 and α_2 , the minimum will again be decided by the D-term in the regime where the F-term and D-terms are comparable. For $g^2\kappa^6 W_{\text{flux}}^2 \gg 1$, the F-term dominates, and the minimum will be decided by whether it is monotonically rising or falling in the regime of validity. Since the J_i in the numerator and denominator are comparable, this rise or fall is essentially flat, and we can set $\alpha_1 = \alpha_2 = 0$. This also matches with the result in the case of global supersymmetry.

The Kahler metric then simplifies into block diagonal form, and in particular the inverse entries in ψ_3, S_1, S_3 space are unaffected by the ψ_1 and ψ_2 , and a direct analytical treatment becomes tractable.

We work in the regime where the F-term dominates over the D-term. Then, the scalar potential becomes (neglecting pure $W_{\text{correction}}$ terms)

$$V \sim e^{\kappa^2 K} \left[\left(-3\kappa^2 + K^{i\bar{j}} \partial_i(\kappa^2 K) \partial_{\bar{j}}(\kappa^2 K) \right) |W_{\text{flux}}|^2 + K^{i\bar{j}} [\partial_i(\kappa^2 K) W_{\text{flux}} \partial_{\bar{j}}(\bar{W}_{\text{correction}}) + cc] \right] \quad (4.21)$$

Taking the open string field V_3 to be stabilized below M_{Planck} we get $|\psi_3| \ll 1$. Also, the regime of validity of the model is $x_3 \gg x_1$.

Evaluating the inverse Kahler metric and keeping to lowest powers of $|\psi_3|$ and x_1/x_3 , one obtains

$$\begin{aligned} e^{\kappa^2 K} \kappa^4 (\partial_{S_1} K)^2 K^{S_1 \bar{S}_1} W_{\text{flux}}^2 &\sim e^{\kappa^2 K} \kappa^2 (.33) W_{\text{flux}}^2, \\ e^{\kappa^2 K} \kappa^4 (\partial_{S_1} K) (\partial_{S_3} K) K^{S_1 \bar{S}_3} W_{\text{flux}}^2 + c.c. &\sim e^{\kappa^2 K} \kappa^2 (.5) W_{\text{flux}}^2, \\ e^{\kappa^2 K} \kappa^4 (\partial_{S_3} K)^2 K^{S_3 \bar{S}_3} W_{\text{flux}}^2 &\sim e^{\kappa^2 K} \kappa^2 (.25) W_{\text{flux}}^2, \end{aligned}$$

while other contributions to $e^{\kappa^2 K} K^{i\bar{j}} \partial_i(\kappa^2 K) \partial_{\bar{j}}(\kappa^2 K) W_{\text{flux}}^2$ contain positive powers of $|\psi_3|$ and x_1/x_3 and are thus further suppressed.

One thus obtains

$$e^{\kappa^2 K} \left(-3\kappa^2 + K^{i\bar{j}} \partial_i(\kappa^2 K) \partial_{\bar{j}}(\kappa^2 K) \right) < 0. \quad (4.22)$$

On the other hand, $K^{i\bar{j}} [\partial_i(\kappa^2 K) W_{\text{flux}} \partial_{\bar{j}}(\bar{W}_{\text{correction}}) + cc]$ gives

$$e^{\kappa^2 K} \left[\Lambda^{7/3} \kappa^{4/3} |\psi_3|^{1/3} e^{-x_1/6} (1 + 2(x_1/x_3) + \dots) \cos(\theta/3 - \text{Im} S_1/3) \right]$$

$$+ \kappa^2 \Lambda_{SO(8)}^3 \alpha e^{-\alpha x_3} x_3 \cos(\alpha \text{Im} S_3) \Big] W_{\text{flux}} \quad (4.23)$$

where θ is the phase of ψ_3 . Setting $\theta/3 - \text{Im} S_1/3 = \pi$ and $\alpha \text{Im} S_3 = \pi$, we get a negative contribution from this term also.

One thus obtains a negative scalar potential in the regime of calculability of the theory. As x_3 and x_1 grow large, $e^{\kappa^2 K} \sim x_3^{-2} x_1^{-1}$ damps out the scalar potential, and V goes to zero. Since the potential is also bounded below as long as the model is well-defined, one obtains an *AdS* minimum. We note that the metastable minimum of the system may lie outside the regime of calculability.

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